Complex roots of the cubic are associated with vector plane rotation

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Abstract

All-real-roots configurations of the univariant cubic equation allow an easy visualization of the roots as the orthogonal projection onto the x-axis of the ends of three root vectors of equal magnitude, radiating symmetrically in the real xy-plane from the origin of the reduced form [Nickalls (1993)].

This Article extends the above three-vector approach to the cubic's complex domain, and shows (a) that the roots of its complex-root configurations can (with some changes) also be interpreted as the orthogonal projection from the ends of symmetrical radiating vectors of equal magnitude onto the real and complex axes, and (b) that this involves a rotation ϕ of the vector plane about the x-axis and into a three-dimensional axis frame.

Since the rotation ϕ is equivalent to an axial rotation about the vector associated with the cubic's real root (aligned with the x-axis), it is consistent with that associated with quartic polynomials [Nickalls (2012)].

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