

Dance of the polynomials



Dick Nickalls

dick@nickalls.org

Department of Anæsthesia

Nottingham University Hospitals

City Hospital Campus, Nottingham, UK

Introduction

Slides showing the cascade of resolvent polynomials associated with the quartic¹ $Z(x) \equiv x^4 - 12x^2 - 10x + y_N = 0$ ($\varepsilon^2 = 2$)

- Blue (solid): quartic $Z(x)$
- Red (solid): Euler's resolvent cubic $R(x)$ of the quartic
- Red (dots): locus $L(x)$ of turning points of resolvent cubic
- Black (vertical dashes): locus of N -point of resolvent cubic — coincides with a turning point of $L(x)$
- Green (solid): resolvent quadratic $Q(x)$ of resolvent cubic
- Green (dots): locus of turning point (Q_N) of resolvent quadratic with respect to the resolvent cubic.
- Magenta (dashes): locus of Q_N with respect to the quartic (is a cubic)

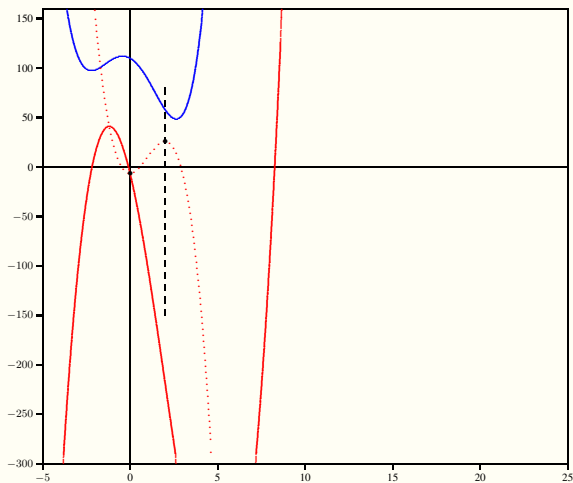
¹Nickalls RWD (2009). The quartic equation: invariants and Euler's solution revealed. *Mathematical Gazette*; 93, 66–75.

Small scale images



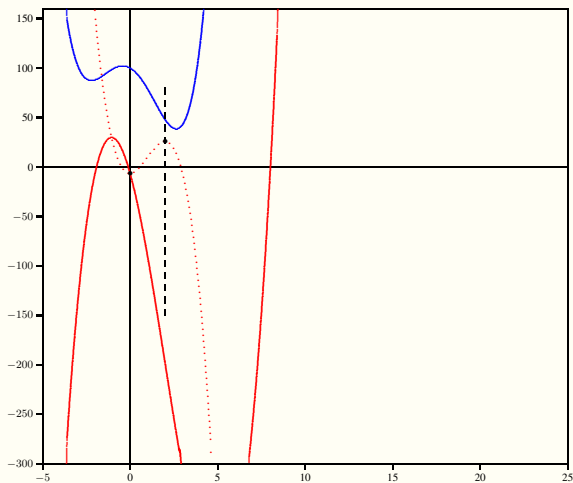
Quartic and related polynomials

quartic $y_N = 110$



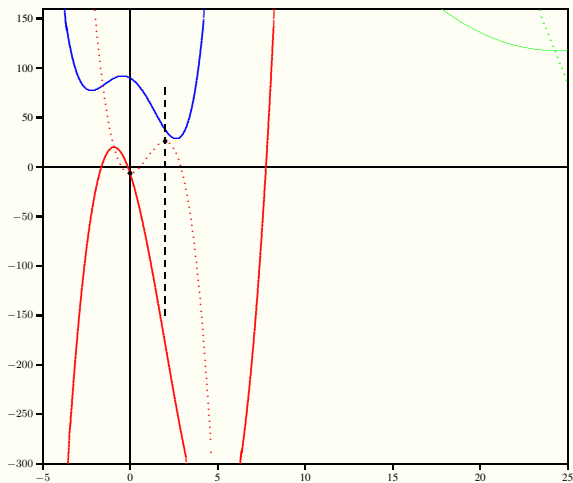
Quartic and related polynomials

quartic $y_N = 100$



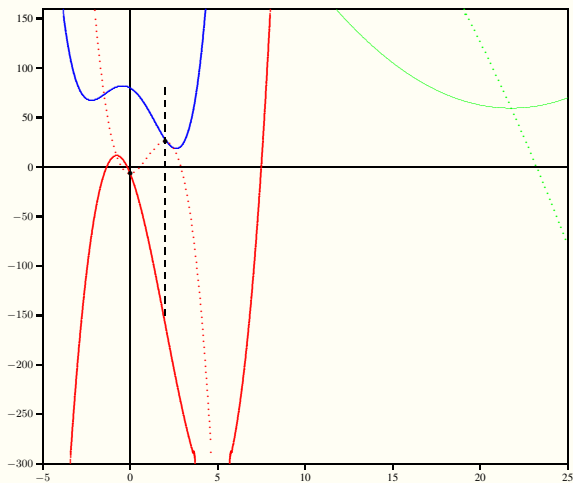
Quartic and related polynomials

quartic $y_N = 90$



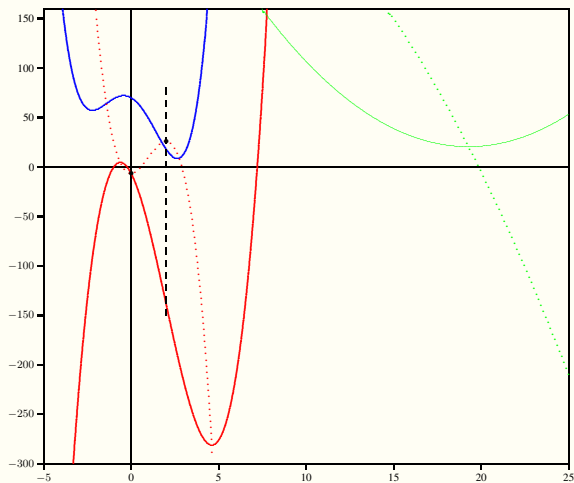
Quartic and related polynomials

quartic $y_N = 80$



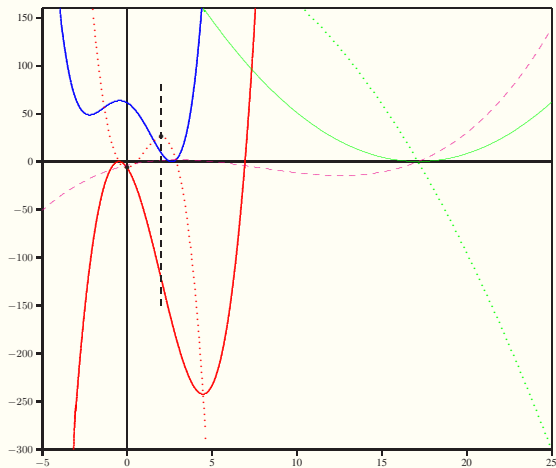
Quartic and related polynomials

quartic $y_N = 70$



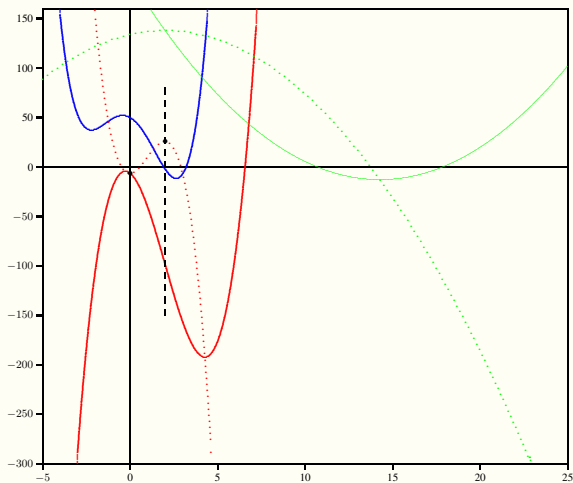
Quartic and related polynomials

double-root condition



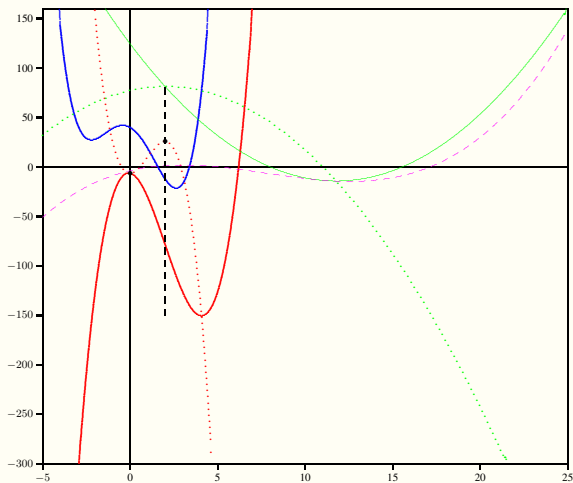
Quartic and related polynomials

quartic $y_N = 50$



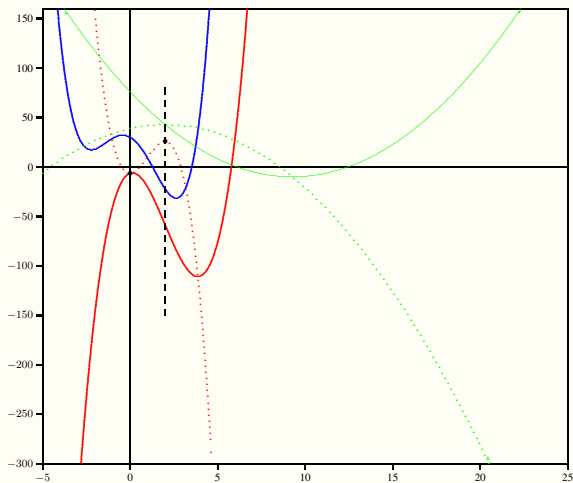
Quartic and related polynomials

quartic $y_N = 40$



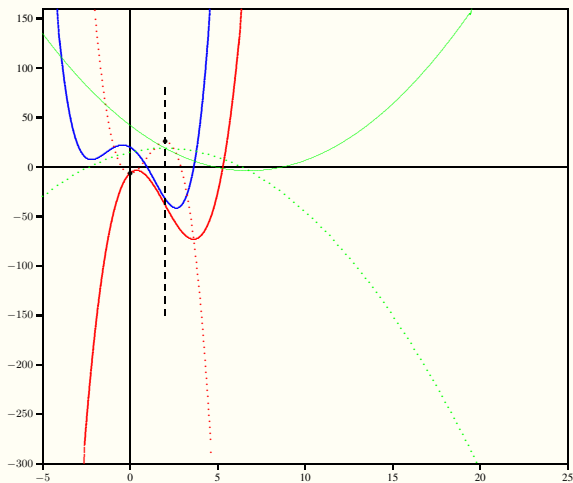
Quartic and related polynomials

quartic $y_N = 30$



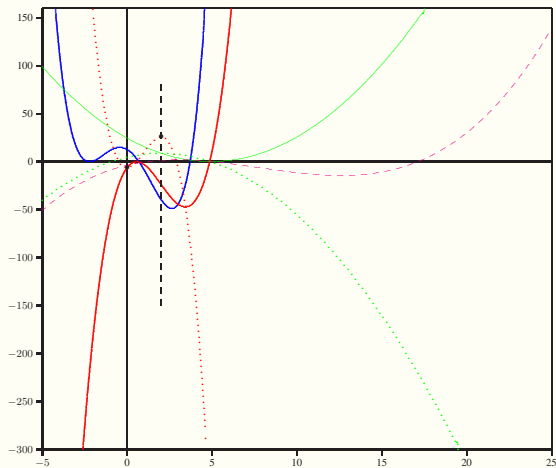
Quartic and related polynomials

quartic $y_N = 20$



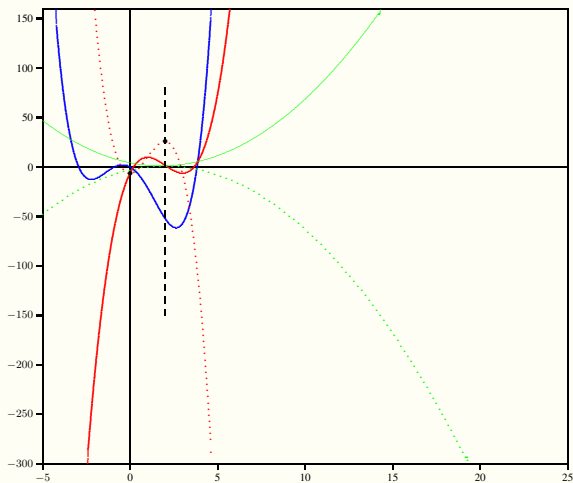
Quartic and related polynomials

double-root condition



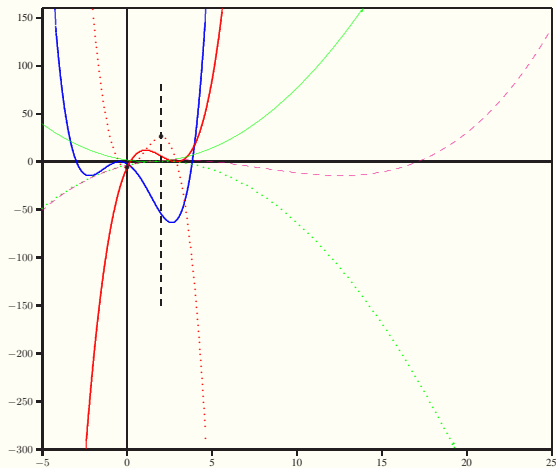
Quartic and related polynomials

quartic $y_N = 0$



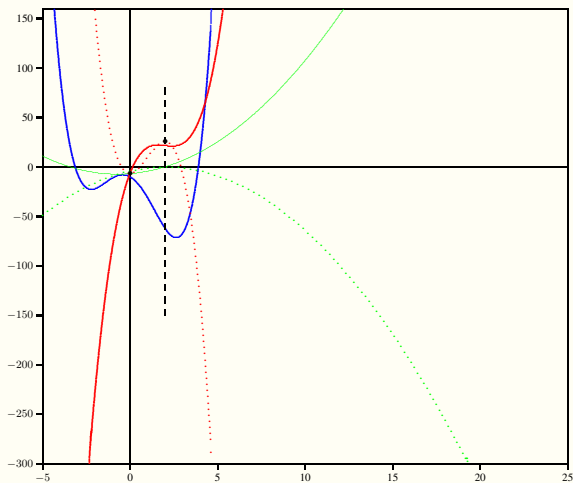
Quartic and related polynomials

double-root condition



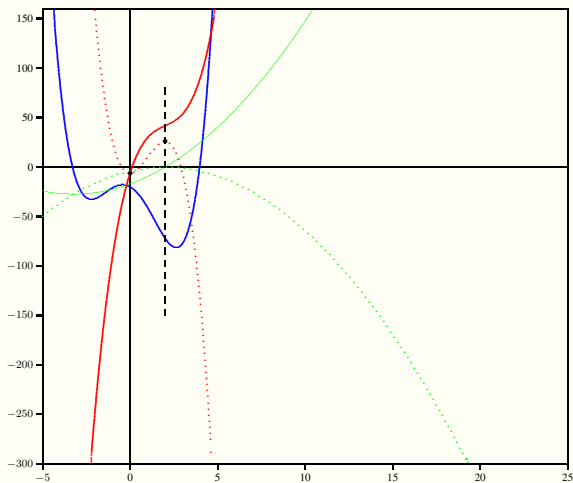
Quartic and related polynomials

quartic $y_N = -10$



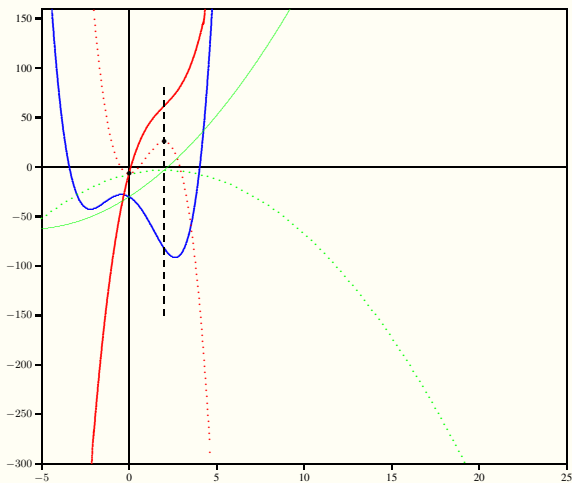
Quartic and related polynomials

quartic $y_N = -20$



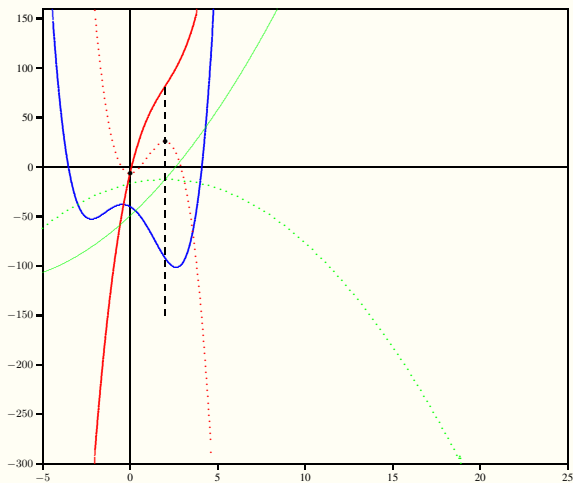
Quartic and related polynomials

quartic $y_N = -30$



Quartic and related polynomials

quartic $y_N = -40$

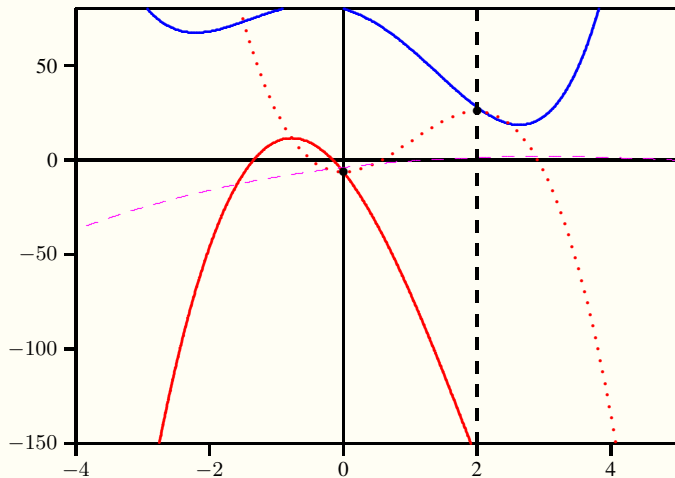


Large scale images



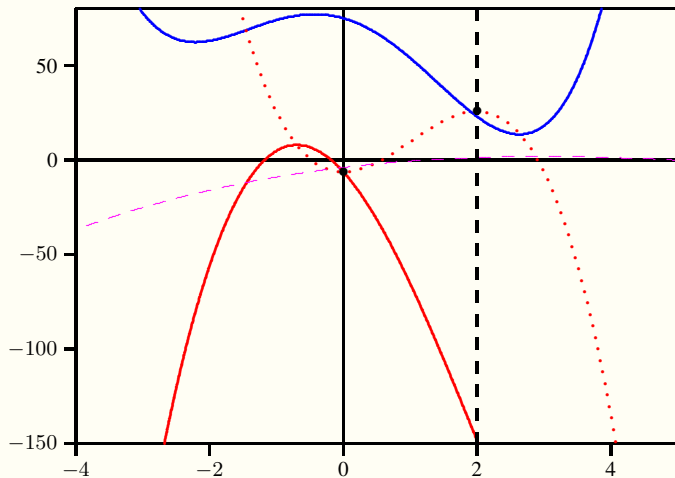
Quartic and related polynomials

quartic $y_N = 80$



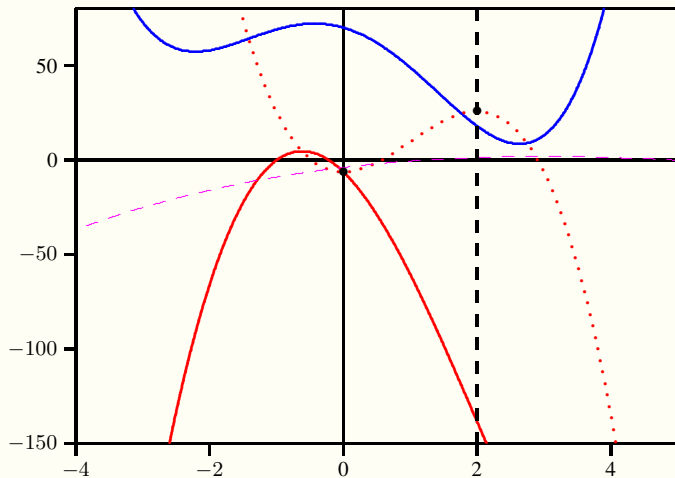
Quartic and related polynomials

quartic $y_N = 75$



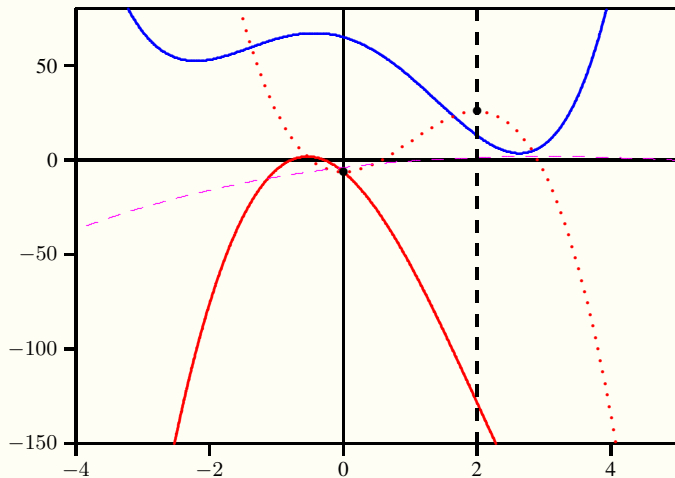
Quartic and related polynomials

quartic $y_N = 70$



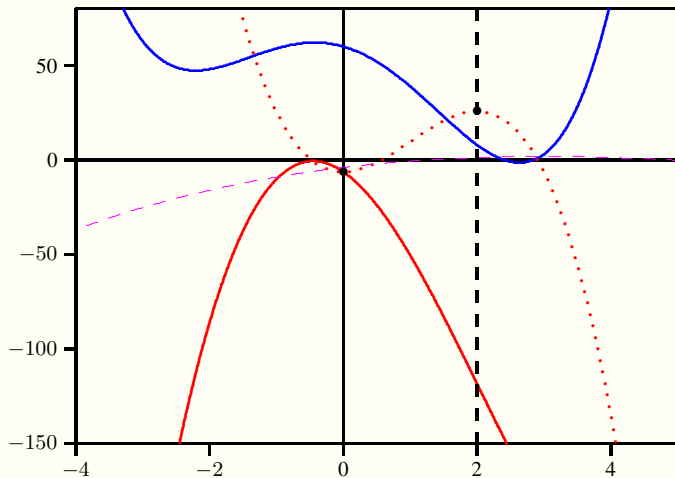
Quartic and related polynomials

quartic $y_N = 65$



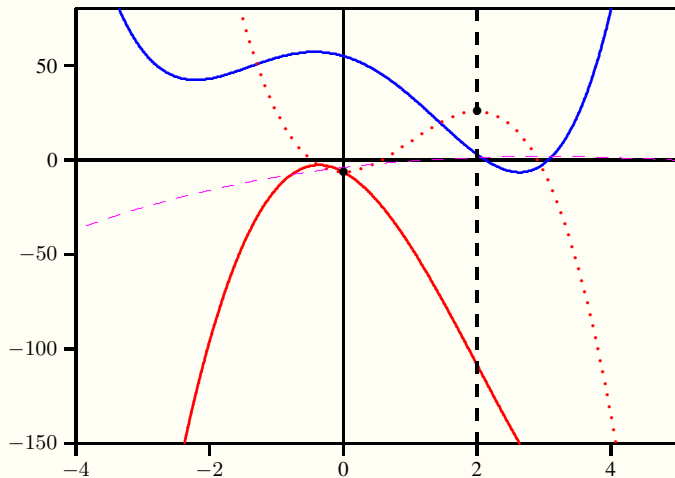
Quartic and related polynomials

quartic $y_N = 60$



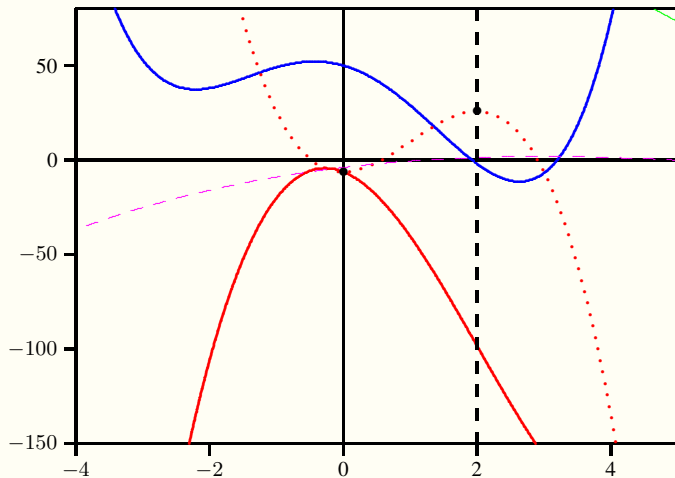
Quartic and related polynomials

quartic $y_N = 55$



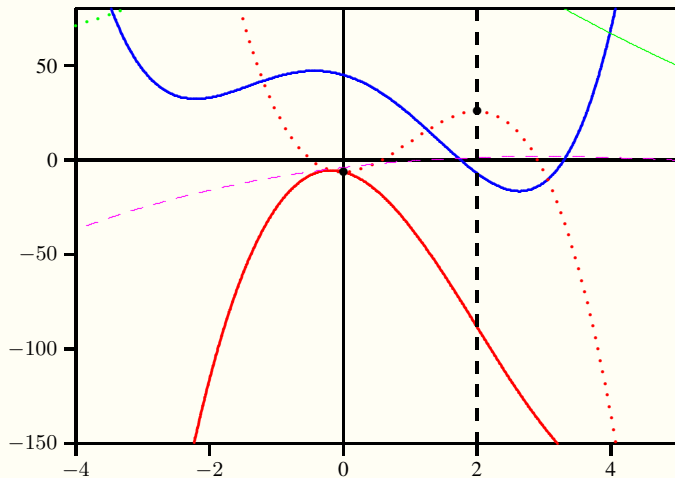
Quartic and related polynomials

quartic $y_N = 50$



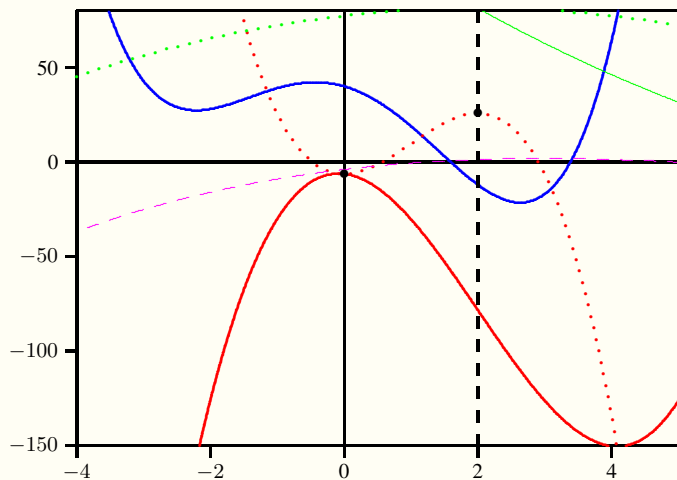
Quartic and related polynomials

quartic $y_N = 45$



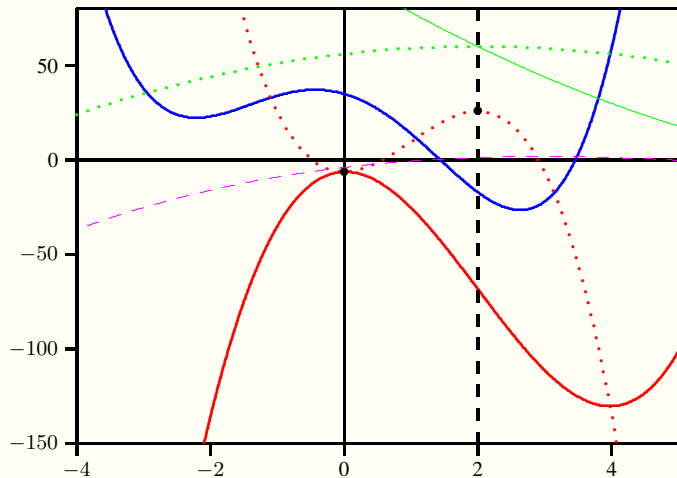
Quartic and related polynomials

quartic $y_N = 40$



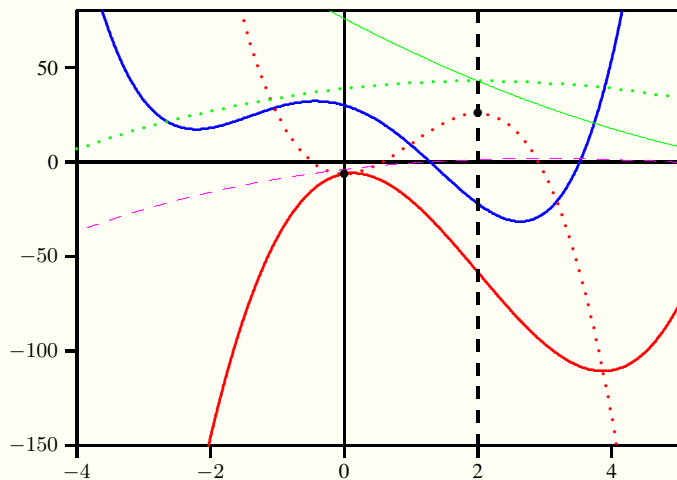
Quartic and related polynomials

quartic $y_N = 35$



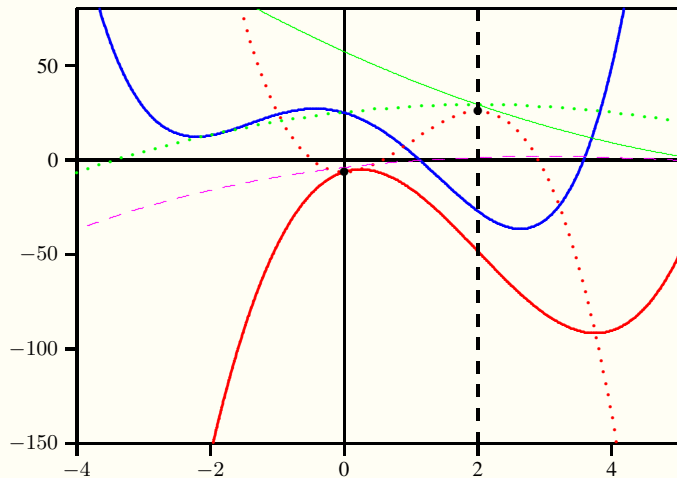
Quartic and related polynomials

quartic $y_N = 30$



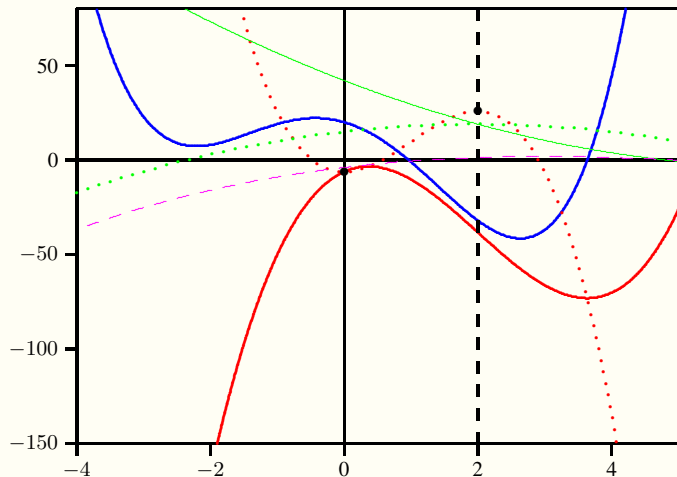
Quartic and related polynomials

quartic $y_N = 25$



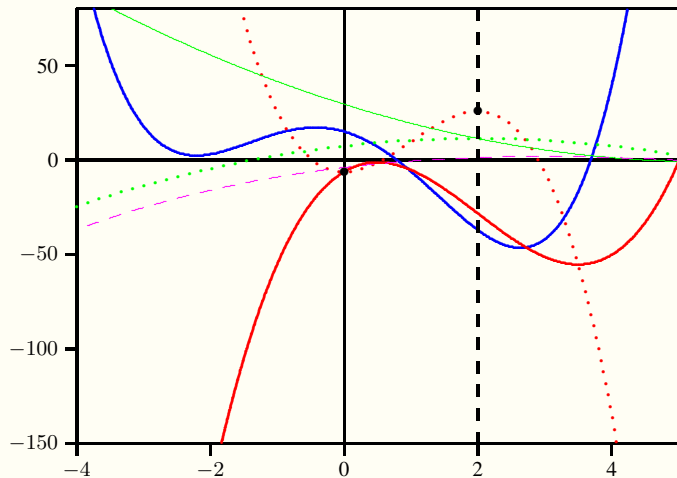
Quartic and related polynomials

quartic $y_N = 20$



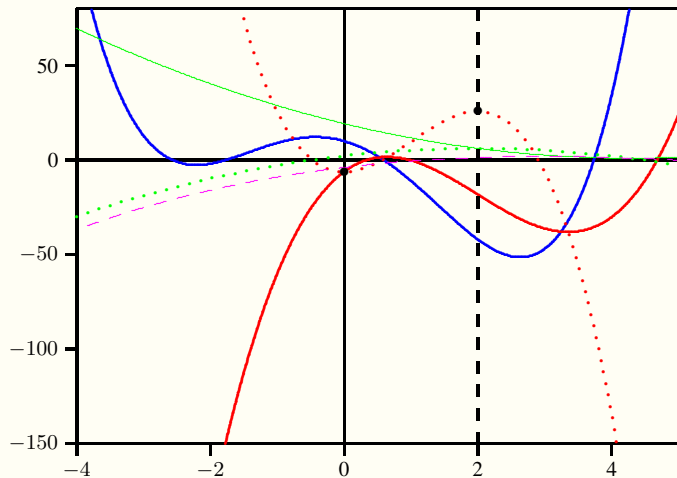
Quartic and related polynomials

quartic $y_N = 15$



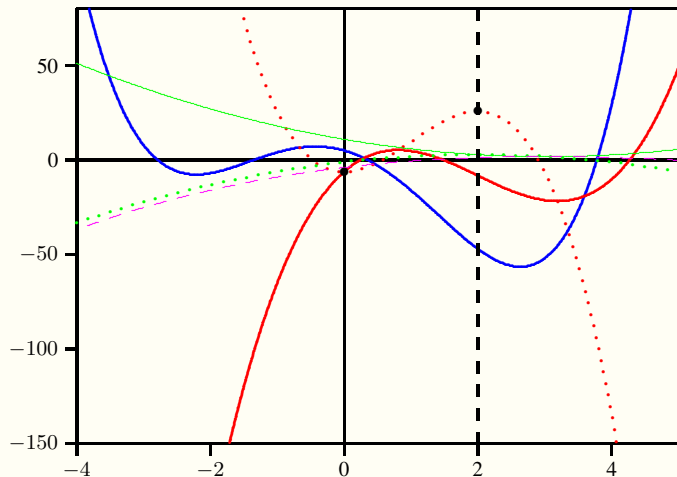
Quartic and related polynomials

quartic $y_N = 10$

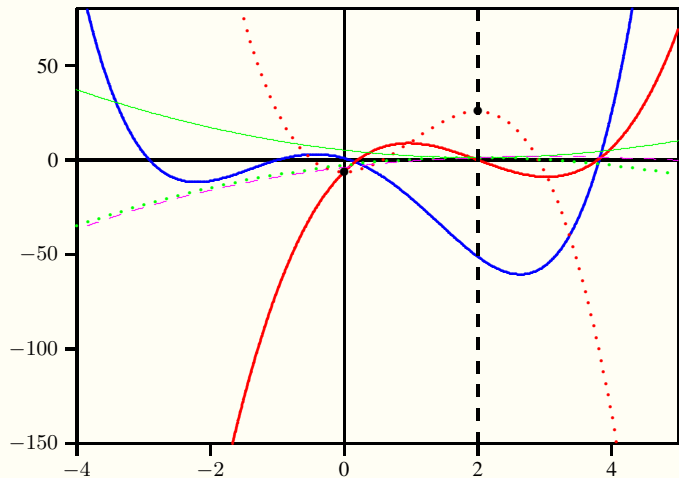


Quartic and related polynomials

quartic $y_N = 5$

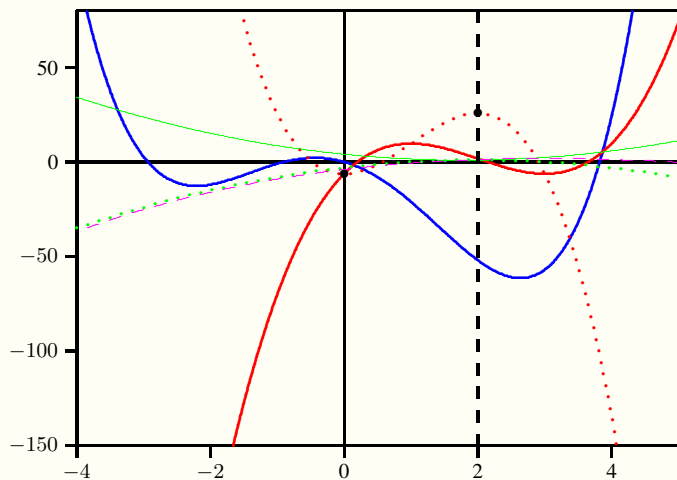


Harmonic condition with four real roots: the N -point of the **resolvent cubic** is on the x -axis; i.e. $J = 0$, $I \neq 0$.



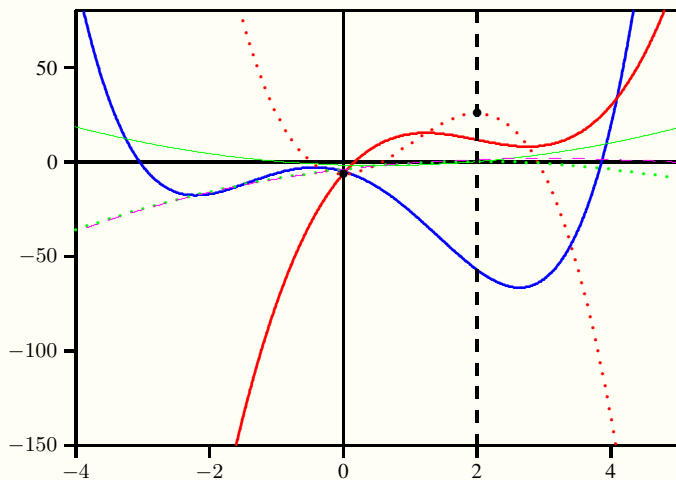
Quartic and related polynomials

quartic $y_N = 0$



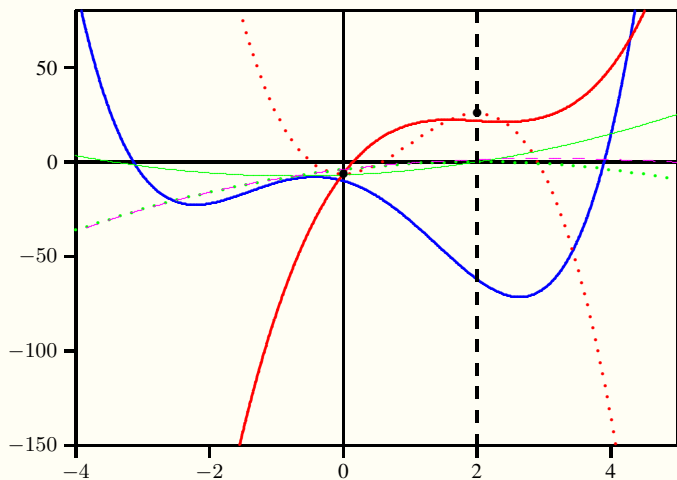
Quartic and related polynomials

quartic $y_N = -5$



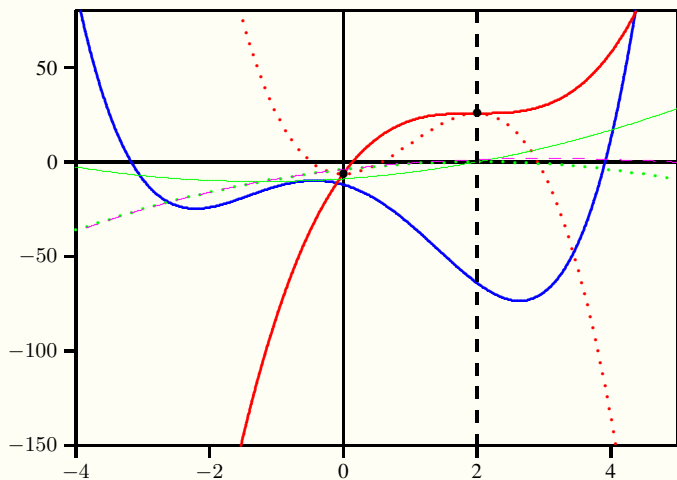
Quartic and related polynomials

quartic $y_N = -10$



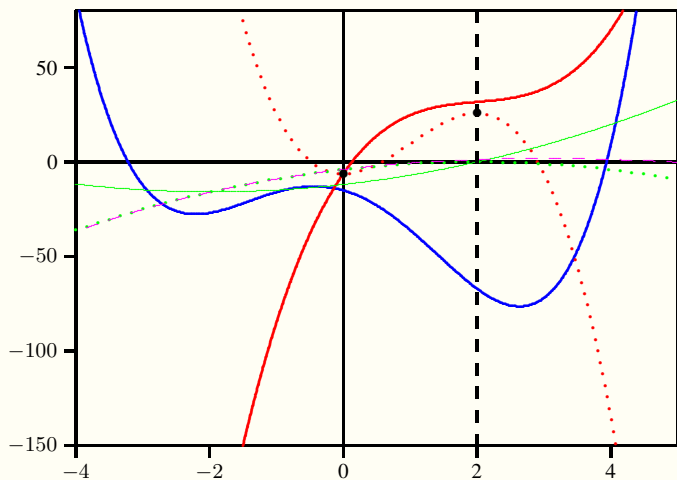
Quartic and related polynomials

quartic $y_N = -12$



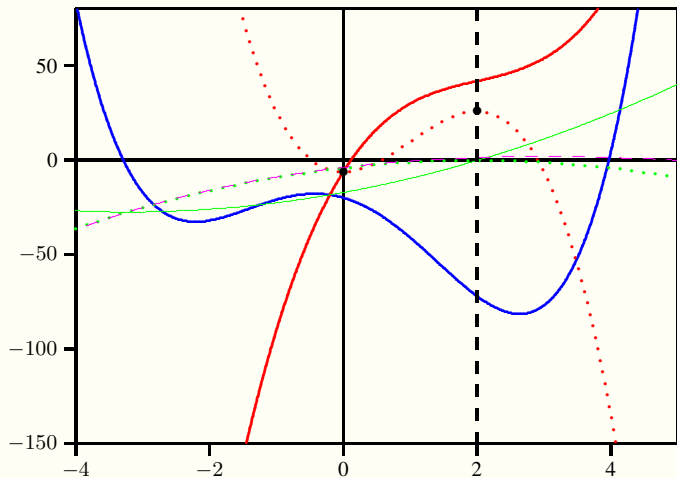
Quartic and related polynomials

quartic $y_N = -15$



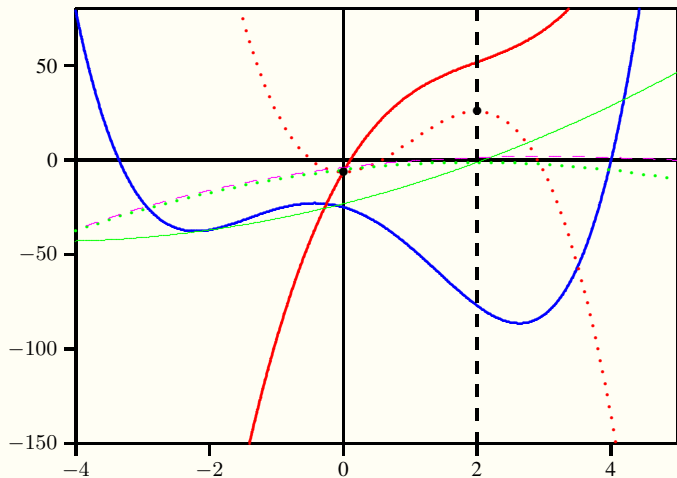
Quartic and related polynomials

quartic $y_N = -20$



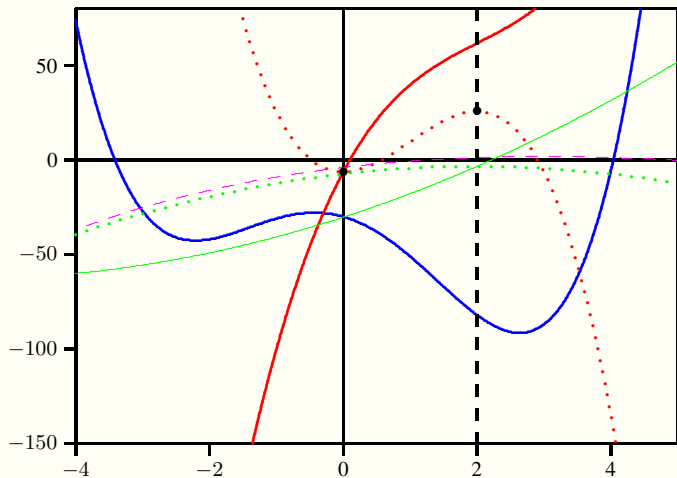
Quartic and related polynomials

quartic $y_N = -25$



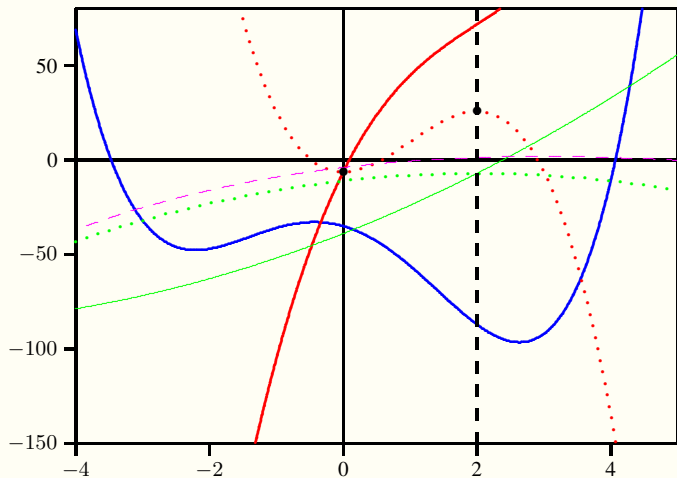
Quartic and related polynomials

quartic $y_N = -30$



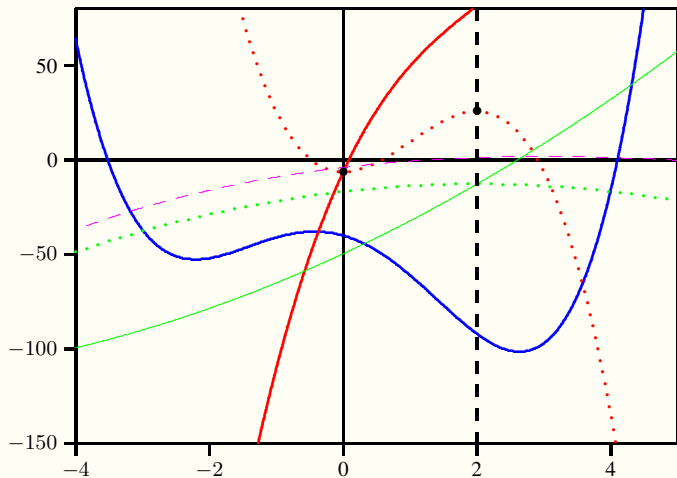
Quartic and related polynomials

quartic $y_N = -35$



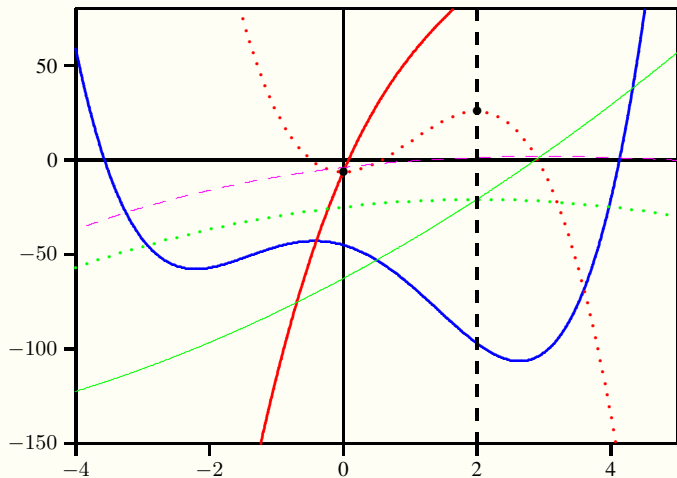
Quartic and related polynomials

quartic $y_N = -40$



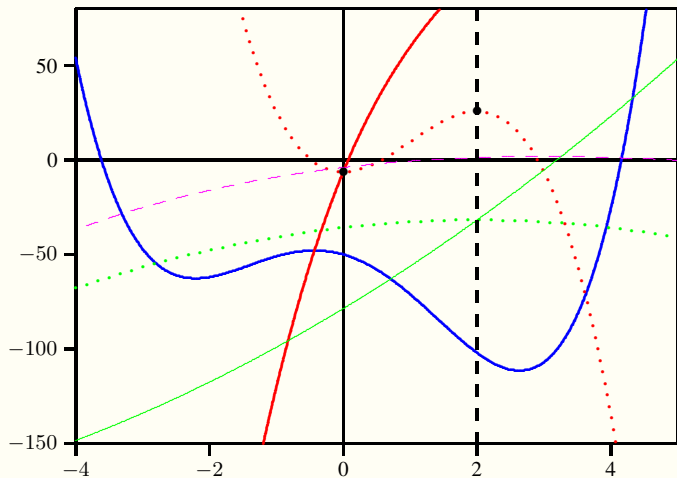
Quartic and related polynomials

quartic $y_N = -45$



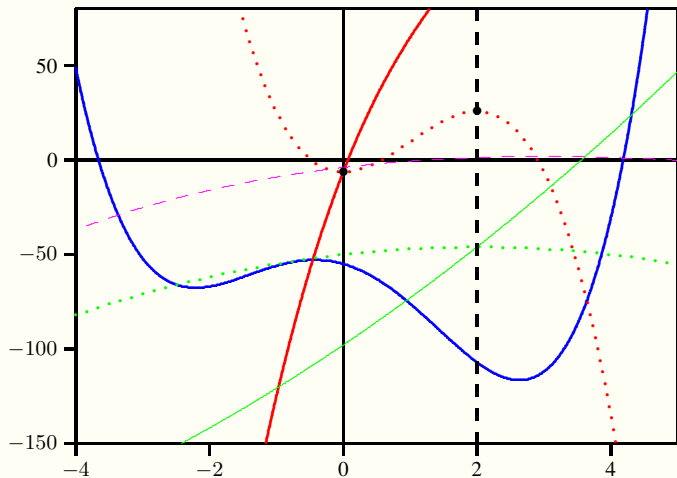
Quartic and related polynomials

quartic $y_N = -50$



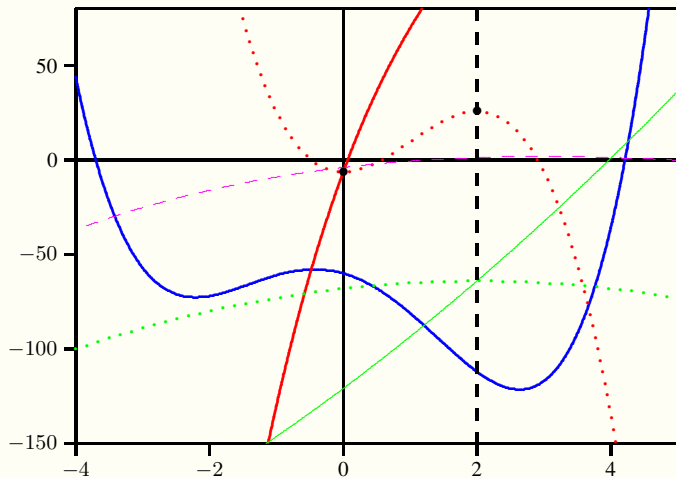
Quartic and related polynomials

quartic $y_N = -55$



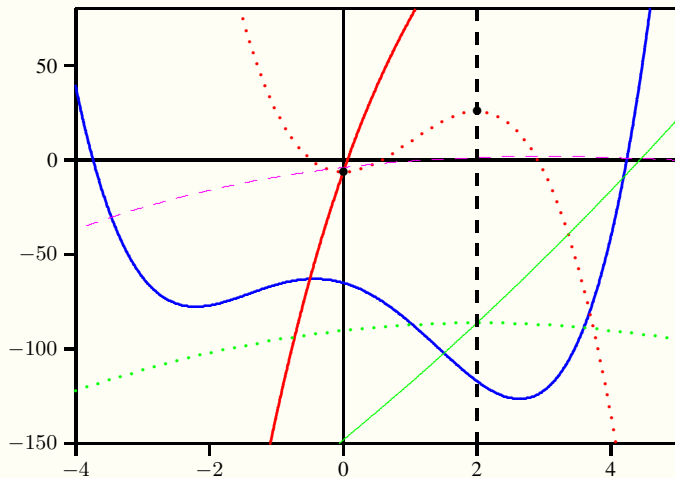
Quartic and related polynomials

quartic $y_N = -60$



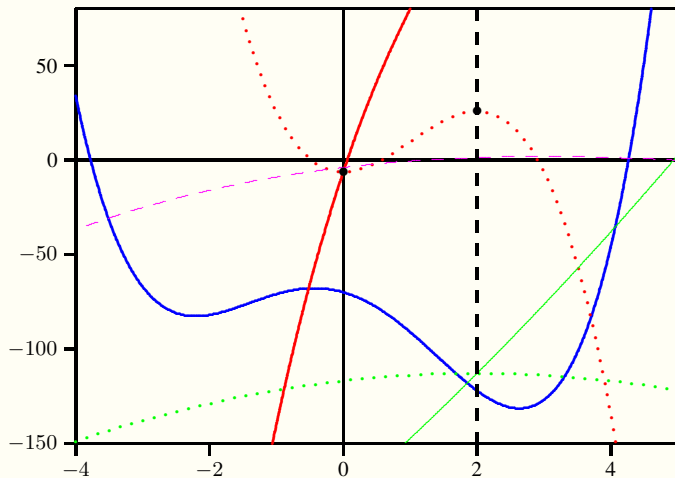
Quartic and related polynomials

quartic $y_N = -65$



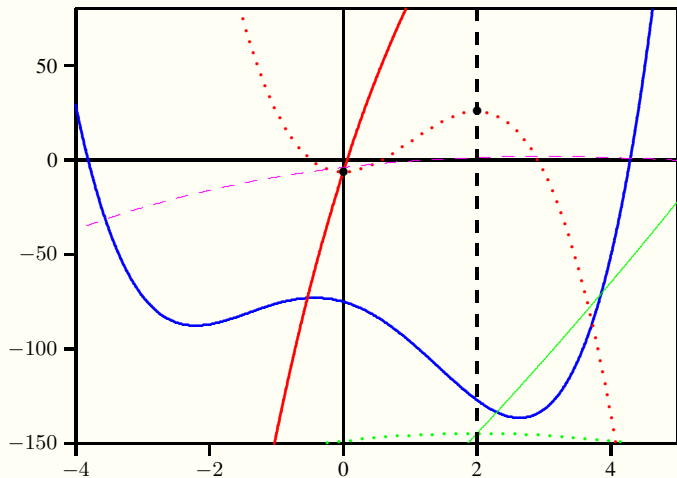
Quartic and related polynomials

quartic $y_N = -70$



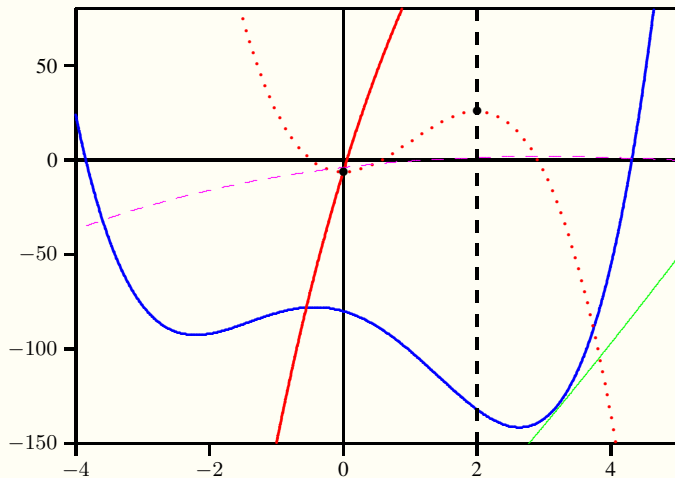
Quartic and related polynomials

quartic $y_N = -75$



Quartic and related polynomials

quartic $y_N = -80$



The end

